TRINITY COLLEGE

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 1&2

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

_____

_____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free   **35% (49 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1   **(6 marks)**

(a)   Determine a unit vector in the same direction as $\mathbf{a} - 3\mathbf{b}$ when $\mathbf{a} = \begin{pmatrix} -7 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$.

(3 marks)

(b)   Determine the value(s) of the constant μ so that the vectors $\begin{pmatrix} \mu + 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} \mu \\ \mu + 3 \end{pmatrix}$ are perpendicular.   (3 marks)

Question 2 **(5 marks)**

Use a vector method to prove that the diagonals OY and XZ of parallelogram $OXYZ$ intersect at right angles if and only if the parallelogram is a rhombus. Let $\overrightarrow{OX} = \mathbf{x}$ and $\overrightarrow{OZ} = \mathbf{z}$.

Question 3 **(8 marks)**

(a) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $R\cos(\theta - \beta)$, where $R > 0$ and $0 < \beta < \dfrac{\pi}{2}$. (3 marks)

(b) Determine the value of $\tan\alpha$, where $90° < \alpha < 180°$, given that α satisfies the equation
$5\cot^2\alpha - \csc^2\alpha = 4\csc\alpha - 2$. (5 marks)

Question 4 **(8 marks)**

(a) Express $z^2 - 4z + 15$ as the product of its linear factors. (3 marks)

(b) Determine the complex numbers u and v given that $u + 2v = 6$ and $u + iv = -1 + 6i$.

(5 marks)

Question 5　　　　　　　　　　　　　　　　　　　　　　　　　**(8 marks)**

(a)　Let $\mathbf{P} = \begin{bmatrix} -2 & 4 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$. Determine $3\mathbf{P} - 4\mathbf{I} + \mathbf{PQ}$.　　　　　(3 marks)

(b)　Determine the value(s) of the constant t for which the matrix $\begin{bmatrix} t+1 & 2 \\ 5 & t-2 \end{bmatrix}$ is not singular.

(2 marks)

(c)　Show use of a matrix method to solve the system of equations $10x + 6y + 3 = 0$
　　and $3x + 2y - 1 = 0$.　　　　　　　　　　　　　　　　　　　　(3 marks)

Question 6   **(6 marks)**

Recall the property $m = \tan\theta$ for the gradient of a straight line making an angle of θ with the positive x-axis, $0 \leq \theta < 90°$ or $90° < \theta \leq 180°$.

(a)   Show that $\dfrac{2m}{1+m^2} = \sin 2\theta$ and that $\dfrac{1-m^2}{1+m^2} = \cos 2\theta$.   **(4 marks)**

(b)   Hence, or otherwise, determine the image of the point with coordinates $(1, -2)$ when it is reflected in the line $y = 2x$.   **(2 marks)**

Question 7   **(8 marks)**

(a)   Express the recurring decimal $0.3\overline{45}$ as a simplified rational number.   (2 marks)

(b)   Let $f(n) = 3^n + 4^n + 5^n$. Prove by induction that $f(n)$ is divisible by 12 for all positive odd integers n.   (6 marks)

Supplementary page

Question number: _____

Supplementary page

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